NAG Toolbox for MATLAB

f04ab

1 Purpose

f04ab calculates the accurate solution of a set of real symmetric positive-definite linear equations with multiple right-hand sides, using a Cholesky factorization and iterative refinement.

2 Syntax

```
[a, c, bb, ifail] = f04ab(a, b, 'n', n, 'm', m)
```

3 Description

Given a set of real linear equations $AX = B$, where A is symmetric positive-definite, f04ab first computes a Cholesky factorization of A as $A = LL^{\mathrm{T}}$, where L is lower triangular. An approximation to X is found by forward and backward substitution. The residual matrix $R = B - AX$ is then calculated using ***additional precision***, and a correction D to X is found by solving $LL^{\mathrm{T}}D = R$. X is replaced by $X + D$, and this iterative refinement of the solution is repeated until full machine accuracy has been obtained.

4 References

Wilkinson J H and Reinsch C 1971 *Handbook for Automatic Computation II, Linear Algebra* Springer–Verlag

5 Parameters

5.1 Compulsory Input Parameters

1: **a(lda,∗) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The upper triangle of the n by n positive-definite symmetric matrix A. The elements of the array below the diagonal need not be set.

2: **b(ldb,∗) – double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{m})$

The n by m right-hand side matrix B.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **m – int32 scalar**

Default: The second dimension of the array **b**.

m, the number of right-hand sides.

Constraint: $\mathbf{m} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb, ldc, wkspce, ldbb

5.4 Output Parameters

1: **a**(**lda,**$*$) **– double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The elements of the array below the diagonal are overwritten; the upper triangle of A is unchanged.

2: **c**(**ldc,**$*$) **– double array**

The first dimension of the array **c** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{m})$

The n by m solution matrix X.

3: **bb**(**ldbb,**$*$) **– double array**

The first dimension of the array **bb** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{m})$

The final n by m residual matrix $R = B - AX$.

4: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

The matrix A is not positive-definite possibly due to rounding errors.

ifail $= 2$

Iterative refinement fails to improve the solution, i.e., the matrix A is too ill-conditioned.

ifail $= 3$

On entry, $\mathbf{n} < 0$,
or $\mathbf{m} < 0$,
or $\mathbf{lda} < \max(1, \mathbf{n})$,
or $\mathbf{ldb} < \max(1, \mathbf{n})$,
or $\mathbf{ldc} < \max(1, \mathbf{n})$,
or $\mathbf{ldbb} < \max(1, \mathbf{n})$.

7 Accuracy

The computed solutions should be correct to full machine accuracy. For a detailed error analysis see page 39 of Wilkinson and Reinsch 1971.

8 Further Comments

The time taken by f04ab is approximately proportional to n^3.

If there is only one right-hand side, it is simpler to use f04as.

9 Example

```
a = [5, 7, 6, 5;
     7, 10, 8, 7;
     6, 8, 10, 9;
     5, 7, 9, 10];
b = [23;
     32;
     33;
     31];
[aOut, c, bb, ifail] = f04ab(a, b)
```

```
aOut =
    5.0000    7.0000    6.0000    5.0000
    3.1305   10.0000    8.0000    7.0000
    2.6833   -0.8944   10.0000    9.0000
    2.2361         0    2.1213   10.0000
c =
     1
     1
     1
     1
bb =
     0
     0
     0
     0
ifail =
           0
```